Modification of the field theory and the dark matter problem

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February 2, 2008

Abstract

We present an extension of the field theory onto the case in which the topology of space can vary. We show that the nontrivial topology of space displays itself in a multivalued nature of all observable fields and the number of fields becomes an additional degree of freedom. In the limit in which topology changes are suppressed, the number of fields is conserved and the Modified Field Theory (MOFT) reduces to the standard field theory where interaction constants undergo an additional renormalization and acquire a dependence on spatial scales. This means that in MOFT particles lose their point-like character and acquire a specific distribution in space, i.e. each point source is surrounded with a halo which carries charges of all sorts. From the dynamical standpoint such halos can be described as the presence of a dark matter or fictitious particles.

When assuming that in the Planck stage of evolution of the Universe topology changes do occur and that the early Universe is in thermodynamic equilibrium, MOFT inevitably predicts the deviation of the law of gravity from Newton's law in a certain range of scales $r_{\min} < r < r_{\max}$, in which the gravitational potential shows essentially logarithmic $\sim \ln r$ (instead of $1/r$) behavior. In this range, the renormalized value of the gravitational constant G increases and at scales $r > r_{\max}$ acquires a new constant value $G' \sim G r_{\max}/r_{\min}$.

We show that in MOFT fermions obey a generalized statistics and at scales $r > r_{\min}$ violate the Pauli principle (more than one fermion can occupy the same quantum state). We also demonstrate that in this range the stable equilibrium state corresponds to the fractal distribution of baryons and, due to the presence of fictitious particles predicted by MOFT, this distribution is consistent with observational limits on $\Delta T/T$. The concept of fictitious particles is then used to explain the origin of the diffuse component in the X-ray background and the origin of Higgs fields. Thus, we show that MOFT allows to relate the rest mass spectrum of elementary particles with cosmological parameters. Finally it is demonstrated that in MOFT, in the range $r_{\min} < r < r_{\max}$ the Universe acquires features of a two-dimensional space whose distribution in the observed 3- dimensional volume has an irregular character. This provides a natural geometric explanation to the observed fractal distribution of galaxies and the

logarithmic behavior of the Newton's potential for a point source. In conclusion we discuss some open problems in MOFT.

1 Introduction

The discrepancy between the luminous matter and the dynamic, or gravitating mass was first identified in clusters of galaxies [1]. Since then it has widely been accepted that the leading contribution to the matter density of the Universe comes from a specific non-baryonic form of matter (unseen, or dark matter, see e.g. Refs. [2, 3]). There are two basic arguments in favor of dark matter and both depend essentially on the underlying theory. First, if the mass distribution in a galaxy follows the brightness the rotation curve is expected to show a Keplerian $r^{-1/2}$ law, while measurements (e.g., see Ref. [4]) show a quite different behavior. Namely, $V(r) \simeq V_c$ stays constant out to the visible edge in most of galaxies. This means that the gravitating mass $M(r)$ contained within a radius r grows as $M(r) = V_c^2 r/G$ and a large fraction of the total mass of a galaxy has a non-luminous dark form. This cannot be any normal form of matter, for the normal matter can always be detected (e.g., intergalactic gas radiate X-rays and is, therefore, seen). Note that there is a number of objects (normal galactic matter - stars, dust, and gas - surrounding a galaxy) which give direct evidence for a spherical halo of dark matter extending far beyond the optical disk of a galaxy [5]. Analysis of the mass-to-light ratio of galaxies, groups, and clusters (e.g., see Ref. [6]) shows that while the M/L ratio of galaxies increases with scale, it flattens and remains approximately constant for groups and clusters. This means that if there is an additional amount of dark matter in clusters (different from that of galactic halos), it should contribute only to the homogeneous background.

The second argument is that the Universe has a rather developed structure (galaxies, clusters, super-clusters). Metric potential fluctuations are measured directly by $\Delta T/T$ in the microwave background [7], and observational values are at least two orders less than it is required by the baryon-dominated Universe. Moreover, it was shown (e.g., see Refs. [8, 9, 10] and for a more recent discussions Refs. [11, 12, 13]) that the observed galaxy distribution exhibits a fractal behavior with dimension $D \approx 2$ which seems to show no evidence of cross-over to homogeneity. Such a picture is in a conflict with the Friedman model unless a substantial amount of dark matter is present to restore the homogeneity of the Universe. Note that this does not remove the conflict with the observed values of $\Delta T/T$, for at the moment of recombination the density of baryons and the CMB temperature are related as $n_b \sim T^3$ and, therefore, the fractal distribution of baryons must leave a direct imprint in the CMB temperature [14].

Apart from some phenomenological properties of the dark matter (e.g., it starts to show up in galactic halos, it is non-baryonic, cold, etc.) nothing is known of its nature. Particle physics suggests various hypothetical candidates for dark matter.

We, however, still do not observe such particles in direct laboratory experiments, while the dark matter displays itself by the gravitational interaction only. Besides, there appears another puzzle that the distribution of the luminous matter traces rather rigidly perturbations in the density of the dark matter (the so-called biased galaxy formation). While such a behavior may be acceptable at the non-linear or quasi-linear stages of the development of perturbations, it looks quite strange and cannot be explained (at least by the presence of hypothetical particles) for the linear stage (e.g., at scales of superclusters where perturbations in the total density are still small $\delta\rho_{tot}/\rho_{tot} \ll 1$). All these facts suggest to try, as an alternative to the dark matter hypothesis, the possibility to interpret the observed discrepancy between luminous and gravitational masses as a violation of the law of gravity.

The best known attempt of such kind is represented by a phenomenological algorithm by Milgrom [15], the so-called MOND (Modified Newtonian Dynamics). This algorithm suggests replacing the Newton's law of gravity, in the low acceleration limit $g \ll a_0$, with $g_{\rm MOND} \sim \sqrt{ga_0}$, where g is the gravitational acceleration and a_0 is a fundamental acceleration constant $a_0 \sim 2 \times 10^{-8} cm/\,s^2$). This, by construction, accounts for the two observational facts: the flat rotation curves of galaxies (for remote stars from the center of a galaxy MOND gives $g_{\rm MOND} = \sqrt{GM_{gal}a_0}/r = V_c^2/r$) and the Tully-Fisher relation $L_{gal} \propto V_c^4$ which gives $M_{gal} \propto L_{gal} \propto V_c^4$ (where L_{gal}, M_{gal}, and V_c are the galaxy's luminosity, mass, and rotation velocity respectively). MOND was shown to be successful in explaining properties of galaxies and clusters of galaxies [16, 17], and different aspects of it still attract attention, e.g., see Refs.[18, 19, 20, 21, 22] and references therein (see also criticism in Ref. [23]).

However, in the present form MOND is not widely accepted, for the lack, in the first place, of a clear theoretical motivation of such a nonlinear behavior from particle physics standpoint (at low accelerations the force $F \propto \sqrt{M}$). Indeed, in particle physics the Newton's law of gravity reflects merely the fact that gravitons are massless particles. There exist processes which are able to violate the standard Newton's law, e.g., vacuum polarization effects are known to produce corrections to the Coulomb potential [24] and analogous corrections are expected to exist for the Newton's potential (e.g., see Ref. [25, 26] where some observational limits on the scale-dependence of the gravitational constant were considered). Such corrections, however, are essential at scales smaller than the Planck length $\ell \lesssim \ell_{pl} \sim 10^{-33} cm$ where quantum gravity effects are believed to be important and this, obviously, differs from the typical scale of a galaxy ($\ell \propto 30 - 50 Kpc \propto 10^{23} cm$) where we would expect the gravity law to violate.

It turned out, however, that quantum gravity effects are, nevertheless, capable of violating the Newton's law at large spatial distances, if possible changes in the topology of space are taken into account (e.g., see Ref. [27]). On the classical level topology changes are known to be forbidden. Such changes were, however, to occur during the quantum period of the evolution of the Universe and, therefore, a relic

of topology transformations may still survive [28]. As it was shown in Ref. [27], the nontrivial topological structure of space displays itself in a renormalization of all interaction constants, which, in general, depends on spatial scales. This does mean the violation of usual interaction laws in some range of scales. Specifically, if on the quantum stage the Universe is assumed to be thermalized with a very high temperature, then one can show [27] that Newton's potential has to transform to essentially logarithmic one ($\sim \ln r$ instead of $1/r$) in a certain range of scales $r_{\min} < r < r_{\max}$. If we identify $r_{\min}$ with the characteristic size of a galaxy, this amplification of the gravity force will produce flat rotation curves without the dark matter hypothesis. We stress that the switch to the logarithmic potential is a rather stringent prediction which does not depend on details of the complete theory (and, in particular, on quantum gravity). In other words, if we believe in topology changes in the very early Universe, we should be very surprised if we would not see a discrepancy between the luminous and dynamical masses.

In fact, the same kind of conclusions holds true for the Coulomb potential and for all other interactions. In this sense we should observe not only dark matter, but dark charges of all sorts as well (and it is very probable that we do).

To illustrate the way how nontrivial topology of space causes a renormalization of charge and mass values we consider a toy example. Let q be the electric charge of a particle in a flat space which includes also a number of handles, and L be the characteristic size of the handles (the distance between end points). In the Coulomb's field of the particle every handle works as a dipole with a moment $d = \delta q L$, for it seizes some fraction of lines of electric force of the particle. It is clear that the farthest end of a handle acquires a charge δq of the same sign as that of the particle, while the nearest end acquires the charge of the opposite sign $-\delta q$. The value δq depends on the distance between the closest end of the handle and the particle and on the characteristic size of the throat of the handle as well (we note that the value δq is always less than q).

Consider now a ball of a radius r with the particle in the center. In general this ball includes a number of ends of the handles, and while $r \ll L$ every handle gets into the ball by one end only. Suppose that there is only one such a handle. Then, the observed value of the charge within the ball of radius $r \ll L$ will be diminished by the value δq, while for $r \gg L$ the ball will include both ends of the handle and the total value of the charge restores. Hence, δq can be interpreted as a dark charge residing between the ends of the handle. In other words every handle transports some fraction of the charge of the particle over a distance of the order L, so a distributed set of handles will create a specific halo of dark charge around every point source. Clearly, we have exactly the same picture for the gravitational potential of the particle, i.e. the handles create the halo of dark mass as well. Properties of the halo depend on the distribution of handles, i.e. on specific properties of the topological structure of space.

Apart from the changes in the gravity law, the qualitative picture shown above

gives at least two direct predictions. First, it can explain the origin of the diffuse component of the X-ray background, for the presence of dark charge extends considerably the characteristic size of a region occupied with plasma (the hot X-ray emitting gas). And secondly, we should expect the existence of fluctuations in the fine structure constant (i.e., in the value of the electric charge). Indeed, if we consider a ball of a radius $r \ll L$ in a moving (with respect to handles) frame, then every handle crossing the ball will cause some variation in the total charge $Q(t) = q - \delta q(t)$ (where $\delta q(t) = 0$ when the handle is outside the ball). This problem, however, requires a more rigorous consideration.

The quantitative description of the situation is somewhat different from the simple picture above. It is based on the suggestion of Ref. [28] that the nontrivial topology of space should display itself in the multivalued nature of all observable fields, i.e. the number of fields should be a dynamical variable. The argument is that in the case of general position an arbitrary quantum state mixes different topologies of space. From the other side, any measurement of such a state should be carried out by a detector which obeys classical laws and, therefore, the detector introduces a background space of a particular topology in terms of which quantum states should be described. It is important that on the classical level the topology is always defined and does not change (i.e., if the Universe was in a particular initial quantum state which mixes different topologies of space, it should eventually remain in the mixed topology state). This means that the topology of space must not be a direct observable, and the only chance to keep the information on the topology is to allow all the fields (which are specified on the background space) to be multivalued. It turns out that a very good compatibility of the multivalued field theory with the conventional one is achieved when the variable number of fields is introduced in the momentum representation, i.e. for Fourier transforms. In other words the topology has to be fixed for the space of momenta, while in the coordinate space it is not defined at all.

The Modified Field Theory (MOFT) which treats multivalued fields was developed in Refs. [28, 27, 29]. Despite MOFT is a self-consistent theory from particle physics standpoint, and basic features and principles of constructing the theory are rather transparent, it is still not sufficiently elaborated. At present we get only particular fragments which we discuss in the present paper, while a satisfactory model of topology transformations is still missing. However the existing fragments seem to be promising and rather impressive. First of all, MOFT provides a natural explanation to the dark matter phenomenon [27]. Secondly, the theoretical scheme of MOFT reconciles the observed developed structure of the Universe (in particular, the fractal distribution of luminous matter) with the homogeneity of the Universe and with the observational limits on $\Delta T/T$ in the microwave background [14]. MOFT also provides a number of new predictions which can be used to verify the theory. In particular, dark charge predicted by MOFT may be responsible for the formation of the diffuse component in the X-ray background, whose nature

seems to be analogous to that of CMB (and very likely for the formation of galactic magnetic fields). Besides, as it is shown in the present paper, MOFT allows to explain the origin of the observed rest mass spectrum of elementary particles and to relate it to cosmological parameters.

2 One-dimensional crystal. Basic ideas of MOFT

We start our consideration with the most simple illustrative example. Consider a one-dimensional crystal. Positions of an atom in the crystal can be described by a single coordinate x. In quantum mechanics states of an atom will be described by a wave function $\psi(x)$. When considering systems with a variable number of atoms the wave function becomes an operator $\widehat{\psi}(x)$ which annihilates (and $\widehat{\psi}^{+}(x)$ creates) one atom at the position x. At low temperatures atoms experience only small oscillations which can be described by a field function $u(x)$ (the function of displacements). This function can be expanded in modes

$$u(x)=\sum_{k}\frac{1}{\sqrt{2\omega(k)L}}\left(a_{k}e^{ikx}+a_{k}^{+}e^{-ikx}\right),$$

where L is the size of the crystal and coefficients a_k^+ and a_k play in quantum theory the role of creation and annihilation operators for phonons. Thus in the leading order the Hamiltonian takes the form

$$H=\sum_{k}\omega(k)\left(a_{k}^{+}a_{k}+\frac{1}{2}\right), \tag{1}$$

where $\omega(k)$ is the energy of a phonon. The ground state corresponds to the zero temperature $T=0$ and represents the vacuum state for phonons, i.e., $a_k|0\rangle=0$, for all k.

Consider now the case of a nontrivial topology of the crystal. To this aim we join two arbitrary points x_1 and x_2 of the crystal, separated by a distance ℓ, with an additional chain of atoms of a length of the order ℓ. The presence of the additional chain of atoms means a degeneracy that appears in the system: more than one atom may have the same position x. In order to describe such a situation we have two possible ways. First one is to change the number of dimensions of the system, i.e., to introduce an additional coordinate y by means of which different atoms, which belong to different chains and have the same position x, could be distinguished. This way presumes that the additional coordinate y is an observable and can be somehow measured. In the example under consideration atoms are 3-dimensional particles and, therefore, y is simply one of the coordinates normal to the line of the crystal. For other degenerate systems, however, y has no such simple interpretation. It can be related to some extra dimensions or to some internal group space.

The second way is to replace the single-valued functions $\psi(x)$ and $u(x)$ in the region $x_1\leq x\leq x_2$ with two-valued functions $\psi^{\alpha}(x)$ and $u^{(\alpha)}(x)$, $\alpha=1,2$. This

can be interpreted as an introduction of the set of two identical fields. The second approach is more general than the first one, because it works also in the case when the extra coordinate y cannot be measured and, therefore, quantum states, which differ in the extra coordinate only, are physically equivalent and should be considered as the same state. In particular, it is exactly the situation which is realized if topology changes in the system are forbidden, for in this case interaction terms in the Hamiltonian do not contain matrix elements which may change the extra coordinate (although this does not mean that the extra coordinate cannot be measured in principle, the removal of the degeneracy requires to invoke additional matter fields which are not a priori contained in the system).

In this manner, we can introduce an operator of the number of fields $N(x)$, which in our example represents the characteristic function $N(x) = 2$ at $x_1 \leq x \leq x_2$ and $N(x) = 1$ at the rest of the points of the crystal. It is important that the number of fields operator depends on quantum states (in our example it is the dependence on x). In a more complicated case $N(x)$ is an arbitrary integer-valued function which characterizes the degree of degeneracy at different values of x or, in other words, the structure of the crystal. The exact form of the structural function depends on conditions which the system was prepared in. Note that such complex systems have a rather wide spread in the nature, e.g. fractal media [30, 31] can be viewed as low-dimensional systems with rather complex structural function $N(x)$. We also point out that in the case of a variable topology (e.g. in the case of percolation systems [32]) the structural function $N(x)$ represents an additional dynamical variable which depends on time.

In the example above the two fields $u^1(x)$ and $u^2(x)$ are identical, for they describe identical atoms. From the mathematical standpoint this means that the Hamiltonian always inherits (from the atoms) the symmetry with respect to permutation $u^\alpha(x) \leftrightarrow u^\beta(x)$. Upon quantizing, fields are described by sets of particles (or quasi-particles). Whether the particles of different sorts (i.e. those which differ in the extra coordinate y) are identical or not, relays heavily on the possibility to measure the extra coordinate. When such measurements are intrinsically forbidden, quantum states, which differ in y only, are physically equivalent, hence respective particles should be considered as indistinguishable. This gives rise to the fact that excitations in such a crystal obey a generalized statistics.

In Ref. [28] an analogous picture was proposed in order to account for possible space-time foam effects. To describe the topology changes during the very early period of the evolution of the Universe one can fancy processes where different pieces of the space constantly re-glued at random points. Once these processes stopped, the structure remains frozen, which means that at the present stage the physical space comes in a number of copies glued together. An equivalent statement is simply that every physical field has to be multivalued. As it was just explained in the example above, once the topology is no longer changing, the fields defined on different sheets of the space have to be taken as indistinguishable. Therefore, the

conclusions made above for quasi-particles in a crystal of a non-trivial topology have to hold true for real particles in an empty space as well. In this sense, MOFT can be viewed as a specialized version of a field theory obeying a generalized statistics. Main principles of the generalized statistics and the generalized second quantization scheme are briefly described in the next section.

3 Generalized second quantization scheme and generalized statistics

Consider a system of identical particles with an undefined a priori symmetry of wave functions. We shall use the Bogoliubov method [33], in which the second quantization is applied to the density matrix (for the case of para-statistics this approach was extended in Ref. [34]). Let us define operators M_{ij} of transitions for particles from a quantum state j into a quantum state i. These operators must obey the Hermitian conditions, i.e.

$$M_{ij}^{+} = M_{ji}, \tag{2}$$

and the algebraic relations expressing the indistinguishability principle for identical particles:

$$[M_{ij}, M_{km}] = \delta_{jk} M_{im} - \delta_{im} M_{kj}. \tag{3}$$

Consider now systems with a variable number of particles. To this end we need to introduce a set of creation and annihilation operators for particles (a_i^+ and a_i) and somehow express the transition operators M_{ij} via them. The simplest generalization of Bose and Fermi statistics was first suggested by H.S. Green [36] and later by D.V. Volkov [37] and is called the parastatistics, or the Green-Volkov statistics.

Consider a set of creation and annihilation operators of particles a_k^+ and a_k, while the transition operators are presented in the form

$$M_{ik} = \frac{1}{2}\left(a_i^+ a_k \pm a_k a_i^+ \mp n_{ik}\right), \tag{4}$$

where n_{ik} is, in general, an arbitrary Hermitian matrix. The upper sign stands for the generalized Bose statistics, while the lower sign stands for the generalized Fermi statistics. The operator $M_i = M_{ii}$ is the operator of the number of particles in the quantum state i. Then the creation and annihilation operators should obey the requirements

$$[M_i, a_k] = -\delta_{ik} a_k, \ \left[M_i, a_k^+\right] = \delta_{ik} a_k^+. \tag{5}$$

Relations (3) and (5) remain invariant under unitary transformations

$$a_i' = \sum_k u_{ik} a_k, \ a_i'^+ = \sum_k u_{ik}^* a_k^+, \tag{6}$$

where $\sum_m u_{im}u^*_{km} = \delta_{ik}$. Thus, applying to (5) an infinitesimal transformation $u_{ik} = \delta_{ik} + \varepsilon\sigma_{ik} + o(\varepsilon)$, where $\sigma^*_{ik} = -\sigma_{ki}$, and retaining the first order terms in ε we get the basic commutation relations for the creation and annihilation operators

$$[M_{kl}, a^+_m] = \delta_{lm}a^+_k, \quad [M_{lk}, a_m] = -\delta_{lm}a_k, \tag{7}$$

which were first suggested by Green [36].

Consider now the vacuum state $|0\rangle$ that is

$$a_k\,|0\rangle = 0 \tag{8}$$

for all k. Then the requirement that for all i and k the transition operators annihilate the vacuum state

$$M_{ik}\,|0\rangle = 0 \tag{9}$$

leads to the condition on one-particle quantum states in the form

$$a_k a^+_i\,|0\rangle = n_{ik}\,|0\rangle\,, \tag{10}$$

which means that the basis of one-particle states is, in general, not orthonormal, but it has norms $\langle 0|\,a_k a^+_i\,|0\rangle = n_{ik}$. From the physical standpoint this signals up the presence of a degeneracy of quantum states (the presence of the extra coordinate in the example of the previous section).

Since n_{ik} is a Hermitian matrix, it follows that it has the diagonal form in a certain basis of one-particle wave functions, i.e. $n_{ik} = \delta_{ik}N_k$ (in the previous section we discussed the case where this matrix has a diagonal form in the coordinate representation). In the simplest case $N_k = N$ is a constant and therefore the relation $n_{ik} = \delta_{ik}N$ remains invariant in an arbitrary basis. Then the condition that norms of vectors in the Fock space are positively defined leads to the requirement that N is an integer number which characterizes the rank of the statistics, or the degree of degeneracy of quantum states [36, 34]. In this simplest case the number N corresponds to the maximal number of particles which admit an antisymmetric (for parabosons) or symmetric (for parafermions) state. The case $N = 1$ corresponds to the standard Bose and Fermi statistics. For the case of a constant rank, Green also gave an ansatz which resolves the relations (7) and (10) in terms of the standard Bose and Fermi creation and annihilation operators

$$a^+_p = \sum_{\alpha=1}^{N} b^{(\alpha)+}_p, \quad a_k = \sum_{\alpha=1}^{N} b^{(\alpha)}_k, \tag{11}$$

where $b^{(\alpha)}_p$ and $b^{(\beta)+}_p$ are the standard Bose (Fermi) operators at $\alpha = \beta$ (i.e. $\left[b^{(\alpha)}_p b^{(\alpha)+}_k\right]_\pm = \delta_{pk}$), but they anti-commutate (commutate) as $\alpha \neq \beta$ (i.e. $\left[b^{(\alpha)}_p b^{(\beta)+}_k\right]_\mp = 0$) for the case of parabose (parafermi) statistics. The presence of an additional index α

in the creation and annihilation operators removes the above mentioned degeneracy of one-particle quantum states.

In the coordinate representation, creation and annihilation of particles is described by the secondly quantized field operators

$$\widehat{\psi}\left(x\right)=\sum\psi_{k}\left(x\right)a_{k},\quad\widehat{\psi}^{+}\left(x\right)=\sum\psi_{k}^{*}\left(x\right)a_{p}^{+},\tag{12}$$

where $\{\psi_k(x)\}$ is a basis of normalized one–particle wave functions. The meaning of these operators is that $\widehat{\psi}^+(x)$ creates (while $\widehat{\psi}(x)$ annihilates) a particle at the point x. In this manner, in the case of parastatistics of a constant rank, field operators $\widehat{\psi}(x,t)$ can be presented in the form $\widehat{\psi}(x,t)=\sum_{\alpha=1}^{M}\widehat{\psi}^{(\alpha)}(x,t)$, i.e. the particles can, in fact, be described by a set of ordinary fields $\{\widehat{\psi}^{(\alpha)}(x,t)\}$, while the many-particle states are classified by the set of occupation numbers $\left|m_k^{(\alpha)}\right\rangle$. In other words the Green representation (11) transforms paraparticles into the set of particles of different sorts, or equivalently into identical particles with an additional internal coordinate. Again, the indistinguishability of particles is here the result of the impossibility of measuring the extra coordinate, which reflects the symmetry of the Hamiltonian with respect to permutations of particles of different sorts. Thus, the states which have the same total occupation numbers $m_k=\sum m_k^{(\alpha)}$ should be considered as physically equivalent states.

One can easily generalize the Green representation (11) onto the case of an arbitrary Hermitian matrix n_{ik}. In the basis in which this matrix takes the diagonal form ($n_{ik}=N_k\delta_{ik}$) the Green representation is given by the same expression (11) in which, however, the rank of statistics N_k depends on the quantum state (the index k). Thus, in the general case, the rank of statistics represents an additional variable. Note that in an arbitrary basis the Green representation does not work and, therefore, we can say that the matrix n_{ik} distinguishes a preferred basis of quantum states. In the previous section we have interpreted the functional dependence of the rank of statistics on the state (and hence the matrix n_{ik}) as a characteristic of the topological structure of the system. Thus, we can say that the topological structure defines a preferred basis of one-particle wave functions for which the classification of quantum states takes the simplest form.

In conclusion of this section we note that from the pure mathematical point of view instead of the Green's ansatz (11) we can use the standard commutation rules for operators $b_p^{(\alpha)}$ and $b_p^{(\beta)+}$, i.e. $\left[b_p^{(\alpha)}b_k^{(\beta)+}\right]_\pm=\delta_{pk}\delta_{\alpha\beta}$. For statistics of a constant rank ($N=const$) this case is trivial, since it corresponds to the usual Bose and Fermi statistics, i.e. $\left[a_ia_k^+\right]_\pm=n_{ik}=N\delta_{ik}$. Still, in applications this case should be considered on an equal footing with the case of parastatistics. Indeed, in the example considered in the previous section, there exist fermionic excitations which obey the standard statistics we just described. For such excitations, according to the Green ansatz, no more than one fermion can occupy the same position x. In the presence of additional links (chains of atoms) it is surely not the general case: if the

crystal has N different links at a position x, then the maximal number of fermionic excitations will also be N (which is the number of different fermionic fields). This means that there will always exist fermionic excitations which obey the parafermi statistics. In real crystals, the degeneracy (which appears due to the presence of the extra coordinate) can be removed (e.g., if processes involving topology transformations are not suppressed), particles with different extra coordinates can become distinguishable and then the presence of a set of fields (instead of a single field) will be essential. It is clear that the same concerns the statistics of bosonic excitations (phonons).

4 Renormalization of interaction constants

In the present section we show that independently on the choice of the statistics of particles the fact that particles in the space of a nontrivial topological structure are described by a set of identical fields results in an additional renormalization of all interaction constants [27]. Indeed, let S be a background basic space and let us specify an arbitrary field φ on it. We suppose that the action for the field can be presented in the following form (for the sake of simplicity we consider the case of linear perturbations only):

$$I=\int\limits_S d^4x\left(-\frac{1}{2}\varphi\widehat{L}\varphi+\alpha J\varphi\right), \tag{13}$$

where $\widehat{L}=\widehat{L}\left(\partial\right)$ is a differential operator (e.g., in the case of a massive scalar field $\widehat{L}\left(\partial\right)=\partial^2+m^2$), J is an external current, which is produced by a set of point sources ($J=\sum J_k\delta\left(x-x_k(s)\right)$, where $x_k\left(s\right)$ is a trajectory of a source), and α is the value of the elementary charge for sources. Thus, the field φ obeys the equation of motion

$$-\widehat{L}\varphi+\alpha J=0. \tag{14}$$

We note that this is valid for perturbations in gauge theories ($\varphi=\delta A_\mu$, where α is the gauge charge) and in gravity ($\varphi=l_{pl}\delta g_{\mu\nu}$, where $\alpha=l_{pl}$ is the Planck length).

In the Modified Field Theory we admit that space have a nontrivial topological structure which cannot be a direct observable. This means that the field φ is not a usual field any more, but is a generalized field which upon quantization gives rise to particles obeying a generalized statistics. It was demonstrated in the previous two sections that the topological structure can be defined by a structural matrix $n\left(x,y\right)$ which determines the degree of the degeneracy of one-particle quantum states. In the basis $\{f_i\}$ in which this matrix takes a diagonal form $n_{ik}=N_k\delta_{ik}$ the nontrivial topological structure is accounted for by the replacement of the field φ_k ($\varphi=\sum_k\varphi_kf_k$) with a set of fields φ_k^a, $a=0,1,...,N_k$.

In particle physics the momentum representation is commonly used (i.e., Fourier transforms $f_k\sim\frac{1}{\sqrt{V}}\exp\left(-ikx\right)$), where the states of the field can be classified in

terms of free particles. In order to allow for the existence of free particles in MOFT, we assume that the matrix n is diagonal exactly in the momentum representation. Moreover, we will show later on that in thermodynamic equilibrium the number of fields N_k in the momentum space is a quite natural object. Thus, the total action assumes the structure

$$I=\int dt\sum_k\sum_{a=0}^{N_k}\left(-\frac{1}{2}\varphi_k^{*a}\widehat{L}_k\varphi_k^a+\alpha J_k^*\varphi_k^a\right). \tag{15}$$

where $\widehat{L}_k=\widehat{L}(\partial_t,-ik)$. Fields φ_k^a are supposed to obey the identity principle and, therefore, they equally interact with the external current.

It is easy to see that the main effect of the introduction of the number of identical fields is the renormalization of the charge (the constant α). To this end we introduce a new set of fields as follows

$$\varphi_k^a=\frac{\widetilde{\varphi}_k}{\sqrt{N_k}}+\delta\varphi_k^a,\quad \sum_a\delta\varphi_k^a=0 \tag{16}$$

where $\widetilde{\varphi}_k$ is the effective ordinary field [28]

$$\widetilde{\varphi}_k=\frac{1}{\sqrt{N_k}}\sum_{a=0}^{N_k}\varphi_k^a. \tag{17}$$

Then the action splits into two parts

$$I=\int dt\left(-\frac{1}{2}\sum_{k,a}\delta\varphi_k^{*a}\widehat{L}_k\delta\varphi_k^a\right)+\int dt\left(-\frac{1}{2}\sum_k\widetilde{\varphi}_k^*\widehat{L}_k\widetilde{\varphi}_k+\sum_k\widetilde{\alpha}_kJ_k^*\widetilde{\varphi}_k\right). \tag{18}$$

The first part represents a set of free fields $\delta\varphi^a$ which are not involved into interactions between particles and, therefore, cannot be directly observed. The second part represents the standard action for the effective field $\widetilde{\varphi}$ with a new value for the charge $\widetilde{\alpha}_k=\sqrt{N_k}\alpha$ which now depends on the wave number k, i.e. it becomes scale-dependent.

We recall that N_k is an operator and we, strictly speaking, should consider an average value for the charge

$$\langle\widetilde{\alpha}\left(k\right)\rangle=\left\langle\sqrt{N_k}\right\rangle\alpha. \tag{19}$$

The requirements of homogeneity and isotropy of the Universe allow $\langle N_k\rangle=N_k\left(t\right)$ to be an arbitrary function of $|k|$. This means that every point source is distributed in space with the density

$$\rho\left(r\right)=\frac{1}{2\pi^2}\int\limits_0^\infty\left(\sqrt{N_k}k^3\right)\frac{\sin\left(kr\right)}{kr}\frac{dk}{k}. \tag{20}$$

If we assume (and we do so) that processes with topology transformations have stopped after the quantum period in the evolution of the Universe, then the structure of the momentum space conserves indeed and the function $\langle N_k \rangle$ depends on time via only the cosmological shift of scales, i.e., $\langle N_k \rangle = N_{k(t)}$, where $k(t) \sim 1/a(t)$ and $a(t)$ is the scale factor. In this manner, function N_k represents some new universal characteristic of the physical space.

5 Description of particles in MOFT

Let ψ be an arbitrary field which, upon the expansion in Fourier modes, is described by a set of creation and annihilation operators $\left\{a_{\alpha,k}, a^+_{\alpha,k}\right\}$, where the index α enumerates polarizations and distinguishes between particles and antiparticles. In what follows, for the sake of simplicity, we ignore the presence of the additional discrete index α. These operators are supposed to satisfy the relations

$$a_k a^+_p \pm a^+_p a_k = \delta_{kp}, \tag{21}$$

where the sign $\pm$ depends on the statistics of particles. In MOFT the number of fields is a variable and, therefore, the set of operators $\left\{a_k, a^+_k\right\}$ is replaced with the expanded set $\left\{a_k(j), a^+_k(j)\right\}$, where $j \in [1, ..., N_k]$. For a free field, the energy is an additive quantity, so it can be written as

$$H_0 = \sum_k \sum_{j=1}^{N_k} \omega_k a^+_k(j)\, a_k(j)\,, \tag{22}$$

where $\omega_k = \sqrt{k^2 + m^2}$. When there is an interaction described by a potential V, the total Hamiltonian $H = H_0 + V$ can be expanded, due to the symmetry with respect to the permutation of particles of different sorts j, in the set of operators [28]

$$A_{m_1,m_2}(k) = \sum_{j=1}^{N_k} \left(a^+_k(j)\right)^{m_1} \left(a_k(j)\right)^{m_2}. \tag{23}$$

For bosonic particles, a single field mode (corresponding to a fixed wave number k) is a quantum mechanical oscillator with a countable set of equidistant energy levels; the level number n corresponds to n bosons with the wave number k. A single fermionic oscillator (i.e. a field mode for fermionic particles) may have only two states, corresponding to $n = 0$ and $n = 1$. In a complete theory with a variable number of fields, quantum states are classified by occupation numbers. To this end, we consider the set of operators $\{C(n,k), C^+(n,k)\}$ which annihilate (resp. create) the oscillators (the field modes) with the wave number equal to k and the number of particles in the mode equal to n. From the mathematical standpoint such operators can be constructed by means of the second quantization of wave

functions for field amplitudes $\Psi(a_k)$. Here it is important that the amplitude a_k represents an observable, i.e. it can be measured. There is no problem with bosonic fields, for amplitudes of bosonic fields are always good observables. The problem appears, however, in the case of fermionic fields, for fermionic amplitudes do not correspond to any observable (all fermionic observables are bilinear combinations in amplitudes). To overcome this difficulty we consider a set of "test" fermionic variables (a set of Grassmanian numbers) ξ_k which obey the relations

$$\xi_k \xi_p + \xi_p \xi_k = 2\delta_{kp} \tag{24}$$

and

$$a_k \xi_p - \xi_p a_k = 0, \tag{25}$$

and construct bilinear combinations $\widetilde{a}_k = \xi_k a_k$. New amplitudes $\widetilde{a}_k$ can already be considered as good observables and we can define operators $\widehat{\Psi}(\widetilde{a}_k)$ and $\widehat{\Psi}^+(\widetilde{a}_k)$ in the standard way. Therefore, in what follows we will understand under fermionic creation and annihilation operators the new variables $\widetilde{a}_k^+$ and $\widetilde{a}_k$, while the standard fermionic operators can be expressed as $a_k = \xi_k \widetilde{a}_k$. With this remark the both cases (fermi and bose fields) can be considered in a unified way.

Thus, the creation and annihilation operators for field modes are supposed to obey the standard relations

$$C(n,k)\, C^+\left(m,k'\right) \pm C^+\left(m,k'\right) C(n,k) = \delta_{nm}\delta_{kk'}\ . \tag{26}$$

and should be used to construct the Fock space in MOFT. The sign $\pm$ in (26) depends on the symmetry of the wave function under field permutations, i.e. on the statistics of fields, which is not the same as the statistics of the particles. Thus, in the case of bosonic fields both signs are possible. E.g., in the example of a one-dimensional crystal discussed earlier, phonons are always bose-particles but the statistics of the phonon fields u^α follows the statistics of atoms. Indeed, phonons represent nothing more than oscillations of atoms with respect to their equilibrium positions. The wave function of the crystal is either symmetric or antisymmetric with respect to permutations of atoms and, therefore, it will be symmetric or antisymmetric with respect to permutations of phonon modes (i.e. permutations of the oscillators whose excitations correspond to the birth of the phonons). In the case of a simple topology of the crystal the question on the symmetry of phonon modes is not essential (all oscillators differ by wave numbers, i.e. there is only one oscillator for every wave number k), while in the case of nontrivial topology a larger than one number of oscillators can correspond to the same wave number k and the type of statistics obeyed by them becomes an issue.

In the case of fermionic fields, however, in order to describe nontrivial topology we should use the Bose statistics for the oscillators. Indeed, let us return to the example of a one-dimensional crystal. In the crystal with a nontrivial topological structure the number of additional links and, therefore, the number of fields required

for the description of excitations is not restricted. Therefore, there should be no restrictions on the number of fermionic excitations at a position x. It is possible only if fermionic field modes obey the Bose statistics, because every such mode can support only one particle (the number of states of a single fermionic oscillator is bounded by two). Of course, from the formal standpoint the case of Fermi statistics for fermionic modes should not be excluded as well. This case, however, corresponds to a standard fermionic field, because the total number of fermionic oscillators will be restricted by $N \leq 2$, and the case $N = 2$ can be identified with $N = 0$ (in both cases no particles can be created).

In terms of C and C^+, operators (23) can be expressed as follows

$$A_{m_1,m_2}(k) = \sum_n \frac{\sqrt{(n+m_1)!\,(n+m_2)!}}{n!} C^+(n+m_1,k)\, C(n+m_2,k) \tag{27}$$

where in the case of bosonic fields the sum is taken over the values $n = 0, 1, ..$, while in the case of fermionic fields $n = 0, 1$, $n + m_1 = 0, 1$ and $n + m_2 = 0, 1$. Thus, the eigenvalues of the Hamiltonian of a free field take the form

$$H_0 = \sum_k \omega_k A_{1,1}(k) = \sum_{k,n\geq 1} n\omega_k N(n,k), \tag{28}$$

where $N(n,k)$ is the number of modes with the wave number k and the number of particles n (i.e., $N(n,k) = C^+(n,k)\, C(n,k)$).

Thus, the field state vector Φ is a function of the occupation numbers $\Phi(N(n,k), t)$, and its evolution is described by the Shrödinger equation

$$i\partial_t \Phi = H\Phi. \tag{29}$$

Consider the operator

$$N_k = A_{0,0}(k) = \sum_n C^+(n,k)\, C(n,k) \tag{30}$$

which characterizes the total number of modes for a fixed wave number k. In standard processes when the number of fields is conserved (e.g., when topology transformations are suppressed) N_k is a constant of motion $[N_k, H] = 0$ and, therefore, this operator can be considered as an ordinary fixed function of the wave numbers.

Consider now the particle creation and annihilation operators. Among the operators $A_{m_1,m_2}(k)$ are some which change the number of particles by one

$$b_m^-(k) = A_{m,m+1}(k), \quad b_m^+(k) = A_{m+1,m}(k), \tag{31}$$

and which replace the standard operators of annihilation and creation of particles, i.e., they satisfy the relations

$$\left[\widehat{n}, b_m^{(\pm)}(k)\right] = \pm b_m^{(\pm)}(k), \quad \left[H_0, b_m^{(\pm)}(k)\right] = \pm\omega_k b_m^{(\pm)}(k), \tag{32}$$

where

$$\widehat{n} = \sum_k \widehat{n}_k = \sum_{k,n} nN(n,k). \tag{33}$$

In the case of fermions there exist only two such operators $b_0^+(k) = C^+(1,k)\,C(0,k)$ and $b_0^-(k) = C^+(0,k)\,C(1,k)$. As we mentioned, proper (para)fermion creation and annihilation operators are given by $\xi_k b_0^+(k)$ and $\xi_k b_0^-(k)$ where ξ_k is a Grassmanian number (see (24)). Then, in the case $N_k = 1$ they transform into the standard fermion creation and annihilation operators.

In the case of bosons the total number of creation/annihilation operators is determined by the structure of the interaction term V. In the simplest case (e.g., in the electrodynamics) the interaction term is expressed solely via $b_0^+(k)$ and $b_0^-(k)$. In this case, we can introduce creation/annihilation operators for the effective field

$$a'_k = \frac{1}{\sqrt{N_k}} b_0^-(k), \quad a'^+_k = \frac{1}{\sqrt{N_k}} b_0^+(k), \tag{34}$$

which satisfy the standard commutation relations, i.e., $\left[a'_k, a'^+_p\right] = \delta_{kp}$. This restores the standard theory, but new features appear, however. First, as it was shown in the previous section, the renormalization (34) results in the renormalization of interaction constants. Secondly, if the fermionic oscillators obey Bose statistics, then in the region of wave numbers in which $N_k > 1$ fermions violate the Pauli principle: up to N_k fermions can be created with the same wave number k.

6 Vacuum state in MOFT

In this section we describe the structure of the vacuum state for bosons and fermions. The true vacuum state in MOFT is defined by the relation

$$C(n,k)\,|0\rangle = 0. \tag{35}$$

In this true vacuum state all modes are absent: $N_k = 0$, hence no particles can be created and all observables related to the field are absent. Thus, the true vacuum state corresponds to the absence of physical space and, in reality, cannot be achieved. Assuming that upon the quantum period of the evolution of the Universe, topology transformations are suppressed, we should require that the number of fields conserves in every mode: $N_k = const$. Then we can define the ground state of the field ψ (which is the vacuum for the particles) as the vector Φ_0 satisfying the relations

$$b_m(k)\,\Phi_0 = 0 \tag{36}$$

for all values k and $m = 0, 1, \ldots$. However, these relations still do not define a unique ground state and should be completed by relations which specify the distribution of modes N_k.

Consider first the case of bosons, and let the bosonic modes obey Fermi statistics (i.e., $\{C(n,k)\,C(m,p)\} = \delta_{nm}\delta_{kp}$). The state Φ_0 corresponds to the minimum energy for a fixed mode distribution $N(k)$. It can be characterized by additional relations

$$b^{+}_{N_k+m}(k)\,\Phi_0 = 0 \quad (m = 0, 1, ...), \tag{37}$$

or, equivalently, by the occupation numbers

$$N(n,k) = \theta(\mu_k - n\omega_k) = \theta(N_k - 1 - n) \tag{38}$$

where $\theta(x)$ is the Heaviside step function and μ_k is the chemical potential which is related to the number of modes N_k as

$$N_k = \sum_n \theta(\mu_k - n\omega_k) = 1 + \left[\frac{\mu_k}{\omega_k}\right]. \tag{39}$$

In particular, from (38) we find that the ground state for bosonic fields contains real particles

$$n_k^* = \sum_{n=0}^{\infty} nN(n,k) = \frac{1}{2}N_k(N_k - 1) \tag{40}$$

and, therefore, corresponds to a finite energy $E_0 = \sum \omega_k n_k^*$. These particles, however, are "dark", for they correspond to the ground state.

In the case of fermionic oscillators which obey Fermi statistics, the occupation numbers in the ground state satisfy (38) as well, with the total number of modes always bounded as $N_k \leq 2$. In the same way as the state $N_k = 0$, the state with $N_k = 2$ can also be considered as a vacuum state (no particles can be created, no annihilated). However, this vacuum also includes hidden particles $n_k^* = \theta(N_k - 2)$.

In the case of Bose statistics for modes, the vacuum state is given by the occupation numbers

$$N(n,k) = N_k\delta_{n,0}, \tag{41}$$

i.e. it is formally constructed from the true vacuum state as follows

$$\Phi_0 = \prod_k \frac{(C^+(0,k))^{N_k}}{\sqrt{N_k!}}\,|0\rangle\,, \tag{42}$$

In contrast to the case of Fermi statistics for modes, the ground state (41) contains no particles and corresponds to the zero energy $E_0 = 0$.

In the case of Fermi particles this ground state satisfies the relation

$$\left(b_0^+(k)\right)^{N_k+1}\Phi_0 = 0. \tag{43}$$

Here, the total number of particles n_k, which can be created at the given wave number k, takes values $n_k = 0, 1, ..., N_k$, i.e. it cannot exceed the number of modes.

In this case the basis of the Fock space consists of vectors of the type

$$|N_k - n_k, n_k\rangle = \prod_k \sqrt{\frac{N_k!}{(N_k - n_k)! n_k!}} \left(b^+ (k)\right)^{n_k} |N_k, 0\rangle \,, \tag{44}$$

where $n_k = 0, 1, ..., N_k$.

Now, assigning a specific value for the function N_k, expressions (38) and (41) define the ground state for respective particles. The function N_k itself cannot be defined within the corresponding field theory. We interpret N_k as a geometric characteristic of the momentum space, which has formed during the quantum period in the evolution of the Universe. Hence, a rigorous derivation of the properties of the function N_k requires studying processes involving topology changes during that period. At the moment, we do not have an exact model describing the formation of N_k and, therefore, our consideration will have a phenomenological character [27, 29].

7 Origin of the spectral number of fields

Assume that upon the quantum period of the evolution of the Universe the matter was thermalized with a very high temperature. Then, as the temperature dropped during the early stage of the evolution, the topological structure of the space (and the spectral number of fields) has tempered and the subsequent evolution resulted only in the cosmological shift of the physical scales.

There exist at least two possibilities. The first and the simplest possibility is the case where processes involving topology changes generate a unique function N_k which is the same for all fields (regardless of their type). However, the mathematical structure of MOFT reserves the more general possibility when the formation of the spectral distribution of modes goes in independent ways for different fields. In this case every particular field ψ_a will be characterized by its own function $N_a(k)$. Which case is realized in the nature can be determined only by confrontation with observations, and below we consider both cases.

Upon the quantum period, the Universe is supposed to be described by the homogeneous metric of the form

$$ds^2 = dt^2 - a^2(t)\, dl^2, \tag{45}$$

where $a(t)$ is the scale factor, and dl^2 is the spatial interval. It is expected that the matter was thermalized with a very high temperature $T > T_{Pl}$ where T_{Pl} is the Planck temperature. Then the state of any field was characterized by the thermal density matrix with mean values for occupation numbers

$$\langle N(k, n)\rangle = \left(\exp\left(\frac{n\omega_k - \mu_k}{T}\right) \pm 1\right)^{-1}, \tag{46}$$

where the signs $\pm$ correspond to the choise of statistics (Fermi or Bose) for the field modes, and the chemical potential μ_k for the given field is related to the spectral number of field modes as

$$N_k = \sum_n \left(\exp\left(\frac{n\omega_k - \mu_k}{T} \right) \pm 1 \right)^{-1}. \tag{47}$$

Consider now the first case when the spectral number of fields N_k is a unique function for all fields. It is well known that near the singularity the evolution of the Universe is governed by a scalar field, while all other fields can be neglected. We assume that the same field is responsible for topology transformation processes which took place in the early Universe. Thus, we can expect that the state of the scalar field was characterized by the thermal density matrix (46) with $\mu = 0$ (for the number of fields varied; strictly speaking, if the field modes satisfy Bose statistics, the chemical potentials cannot vanish and have to take some rest value – one can choose $\mu_k = \omega_k/2$). On the early stage $m \ll T$, and the temperature and the energy of scalar particles decreased with time proportionally to $a(t)^{-1}$. When the temperature droped below a critical value T_*, which corresponds to the moment $t_* \sim t_{pl}$, topological structure (and the number of fields) tempered. This generated (see (46)) the value $N_k \sim T_*/\omega_k$ for the case of Fermi statistics of the oscillators of the scalar field, or $N_k \sim \ln(T_*/\omega_k)T_*/\omega_k$ in the Bose statistics case. The logarithmic factor will not influence the further results in a noticeable way, so we will further stick to the Fermi statistics case, for simplicity.

Thus, the value of N_k become frozen as $N_k \sim T_*/\omega_k$ at $t \sim t_*$, and (47) defines the chemical potential for the scalar field as $\mu \sim T_*$, a constant for all k. Let us neglect the temperature corrections, which are essential only at $t \sim t_*$ and whose role is in smoothing the real distribution N_k. Then at the moment $t \sim t_*$ the ground state of the scalar field will be described by (38) with $\mu_k = \mu = const \sim T_*$. During the subsequent evolution, the physical scales are subjected to the cosmological shift, however the form of this distribution in the comoving frame must remain the same. Thus, on the later stages $t \geq t_*$, we find (see (39)):

$$N_k = 1 + \left[\frac{\widetilde{k}_1}{\Omega_k(t)} \right], \tag{48}$$

where $\Omega_k(t) = \sqrt{a^2(t)\, k^2 + \widetilde{k}_2^2}$, $\widetilde{k}_1 \sim a_0\mu$, and $\widetilde{k}_2 \sim a_0 m$ ($a_0 = a(t_*)$). From (48) we see that there is a finite interval of wave numbers $k \in [k_{\min}(t), k_{\max}(t)]$ on which the number of fields N_k changes its value from $N_k = 1$ (at the point $k_{\max}$) to the maximal value $N_{\max} = 1 + \left[\widetilde{k}_1/\widetilde{k}_2\right]$ (at the point $k_{\min}$). The boundary points of the interval of k depend on time and are expressed via the free phenomenological parameters $\widetilde{k}_1$ and $\widetilde{k}_2$ as follows

$$k_{\max} = \frac{1}{a(t)} \sqrt{\widetilde{k}_1^2 - \widetilde{k}_2^2}, \quad k_{\min} = \frac{1}{a(t)} \sqrt{\widetilde{k}_1^2 / (N_{\max} - 1)^2 - \widetilde{k}_2^2}. \tag{49}$$

Out of this interval, the number of fields remains constant i.e., $N_k = N_{\max}$ for the range $k \leq k_{\min}(t)$ and $N_k = 1$ for the range $k \geq k_{\max}(t)$. From restrictions on parameters of inflationary scenarios we get $m \lesssim 10^{-5} m_{Pl}$ which gives $N_{\max} \gtrsim 10^5 T_*/m_{pl}$, where T_* is the critical temperature at which topology has been tempered. Since we assume that the numbers N_k are the same for all fields, we can now substitute (48) in (47) and find the corresponding values of the chemical potentials μ_k for all other particles.

Consider now the second case when the spectral number of modes N_k forms independently for different fields. We choose Fermi statistics for field oscillators. In this case bosonic fields are described by the same distribution (48) in which, however, the parameters $\widetilde{k}_1$ and $\widetilde{k}_2$ are free phenomenological parameters which are specific for every particular field. Thus, for massless fields we find $\widetilde{k}_2 = k_{\min} = 0$ and $N_k = 1 + [k_{\max}/k]$. In the case of fermions the chemical potential μ_k cannot vanish and for $T \geq T_*$ it should take some rest value $\mu_k = \epsilon_0$. Thus, in the same way as in the case of bosons, we find $N_k = 1 + \theta(k_{\max} - k)$, where $\theta(x)$ is the Heaviside step function and $k_{\max} \sim T_* a(t_*)/a(t)$. In this case the spectral number of fermions is characterized by the only phenomenological parameter and $N_k = N_{\max} = 2$ as $k < k_{\max}$.

As we see, the properties of the spectral number of fields can be different, depending on which case is realized in the nature. We note, however, that if in the first case the spectral number of fields N_k can be considered as a new geometric characteristics which straightforwardly defines properties of the space and hence of all matter fields, in the second case we, strictly speaking, cannot use such an interpretation. Moreover, if the last case is really realized in the nature, it should relate to yet unknown processes. Therefore, in the next sections we will discuss the first possibility only.

In conclusion of this section we note that the real distribution can be different from (48), which depends on the specific picture of topology transformations in the early Universe and requires the construction of the exact theory (in particular, thermal corrections smoothen the step-like distribution (48)). However we believe that the general features of N_k will remain the same.

8 The law of gravity

The dependence of charge values upon wave numbers means that particles lose their point-like character and this leads to the fact that the standard expressions for Newton's and Coulomb's energy of interaction between particles break down. In this section we consider corrections to Newton's law of gravity (corrections to Coulomb's law are identical). Here, the interaction constant $\alpha \sim m\sqrt{G}$ (where m is the mass of a particle), and MOFT gives $G \to G(k) = N_k G$. To make estimates, we note that at the moment $t \sim t_*$ the mass of scalar particles should be small as compared with the chemical potential (which has the order of the Planck energy),

which gives $\widetilde{k}_1 \gg \widetilde{k}_2$ in (48). Then, in the range $k_{\max}(t) \geq k \gg k_{\min}(t)$ the function N_k can be approximated by

$$N_k \sim 1 + [k_{\max}(t)/k]\,. \tag{50}$$

Consider two rest point particles with masses m_1 and m_2. Then the Fourier transform for the energy of the gravitational interaction between particles is given by the expression

$$V(\mathbf{k}) = -\frac{4\pi G m_1 m_2}{|\mathbf{k}|^2} N_k. \tag{51}$$

The coordinate representation is given by the integral

$$V(r) = \frac{1}{2\pi^2}\int\limits_0^\infty V(\omega)\,\omega^3 \frac{\sin(\omega r)}{\omega r}\frac{d\omega}{\omega}. \tag{52}$$

From (48) and (49) we find that this integral can be presented in the form

$$V(r) = -\frac{2Gm_1m_2}{\pi}\sum_{n=0}^{N_{\max}-1}\int\limits_0^{k_n}\frac{\sin(\omega r)}{\omega r}d\omega = -\frac{Gm_1m_2}{r}\left(1+\sum_{n=1}^{N_{\max}-1}\frac{2Si(k_n r)}{\pi}\right) \tag{53}$$

where $k_n = \frac{1}{a(t)n}\sqrt{\widetilde{k}_1^2 - n^2\widetilde{k}_2^2}$. The first term (with $n=0$) of the sum in (53) gives the standard expression for Newton's law of gravity, while the terms with $n>1$ describe corrections. In the range $k_1 r = k_{\max} r \ll 1$ we have $Si(k_n r) \sim k_n r$, and corrections to Newton's potential give a constant

$$\delta V \sim -\frac{2Gm_1m_2}{\pi}\sum_{n=1}^{N_{\max}-1} k_n. \tag{54}$$

Thus, in this range we have the standard Newton's force. In the range $k_{\min} r \gg 1$, we get $\frac{2}{\pi}Si(k_n r) \sim 1$, and for the energy (53) we find

$$V(r) \sim -\frac{G' m_1 m_2}{r},$$

where $G' = GN_{\max}$. Thus, on scales $r \gg 1/k_{\min}$ the Newton's law is restored, however the gravitational constant increases in $N_{\max}$ times. In the intermediate range $1/k_{\min} \gg r \gg 1/k_{\max}$ the corrections can be approximated as

$$\delta V(r) \sim \frac{2Gm_1m_2}{\pi}\frac{\widetilde{k}_1}{a(t)}\ln\left(\frac{\widetilde{k}_2}{a(t)}r\right), \tag{55}$$

i.e., they have a logarithmic behavior.

We note, that from the dynamical point of view the modification of the Newton's law of gravity can be interpreted as if sources acquire an additional distribution in space. Indeed, let m_1 be a test particle which moves in the gravitational field created by a source m_2. Then, assuming Newton's law is unchanged and the test particle is point-like, from (53) we conclude that the source m_2 is distributed in space with the dynamical density

$$\rho_{dyn}(r) = \frac{m_2}{2\pi^2}\int\limits_0^\infty N_k k^3 \frac{\sin(kr)}{kr}\frac{dk}{k} = m_2\left(\delta(\vec{r}) + \frac{1}{2\pi^2}\sum_{n=1}^{N_{\max}-1}\frac{\sin(k_n r) - k_n r\cos(k_n r)}{r^3}\right). \tag{56}$$

Then the total dynamical mass contained within a radius r is

$$M_{dyn}(r) = 4\pi\int_0^r s^2\rho(s)\,ds = m_2\left(1 + \frac{2}{\pi}\sum_{n=1}^{N_{\max}-1}\left(Si(k_n r) - \sin(k_n r)\right)\right). \tag{57}$$

We stress that this dynamical mass accounts for both distributions (i.e., the actual distribution of the source and that of the test particle). Thus, in the range $r \ll 1/k_{\max}$ we find $M_{dyn}(r) \sim m_2$, i.e., one may conclude that the gravitational field is created by a point source with the mass m_2. However in the range $1/k_{\min} > r > 1/k_{\max}$ the dynamical mass increases as $M_{dyn}(r) \sim m_2 k_{\max} r$, and for $r \gg 1/k_{\min}$ the mass reaches the value $M_{dyn}(r) \sim m_2 k_{\max}/k_{\min}$. This is in a very good agreement with what rotation curves in galaxies show [4]. We note that the same formulas (56) and (57) work in the case of charged particles as well, with the obvious substitution $m\sqrt{G} \to e$, which gives rise to the concept of the dynamical charge.

In this manner we see that in MOFT the distributions of the dark matter and the actual matter are strongly correlated (by the rule (56)), and the resulting behavior of the dynamically determined mass $M(r)$ agrees with the observation on the scale of galaxies. We stress that the theoretical scheme of MOFT was not invented to fit the dark matter distribution. On the contrary, the logarithmic behavior of the effective field potentials simply appears in the thermodynamically equilibrium state at the low temperature, as a by-product of a non-trivial structure of MOFT vacuum.

9 Dark matter or fictitious baryons

We see that in the case when the number of fields is conserved MOFT reduces to the standard field theory in which interaction constants undergo a renormalization and, in general, acquire a dependence on spatial scales. From the physical standpoint such a renormalization means that particles lose their point-like character and acquire a specific distribution in space, i.e. each point particle is surrounded with an additional halo. This halo carries charges of all sorts and its distribution around a point source follows properties of the vacuum state in MOFT (i.e. of the topological structure of space).

For applied problems the presence of additional halos can be accounted for by the formal replacement of interaction constants α with operators $\widehat{\alpha}\left(-i\nabla\right)$ (where $\nabla = \partial/\partial x$). In the reference system at rest, which is distinguished by the microwave background, the Fourier transforms of these operators are given by the expression $\widetilde{\alpha}\left(k\right) = \sqrt{N_k}\alpha$ with a universal structural function N_k. In the simplest case this function can be taken from (48) and it can be described by two phenomenological parameters which represent the two characteristic scales. They are the minimal scale $r_{\min} = 2\pi/k_{\max}$ on which the dark matter starts to show up (and on which the law of gravity (53) starts to deviate from Newton's law) and the maximal scale $r_{\max} = 2\pi/k_{\min}$ which defines the fraction of the dark matter or the total increase of interaction constants $G_{\max} = Gr_{\max}/r_{\min}$ (and after which the Newton's law restores). Both scales depend on time via the cosmological shift of scales $r\left(t\right) = a\left(t\right)r$. The minimal scale $r_{\min}$ can be easily estimated (e.g., see Ref.[38]) and constitutes a few *kpc*. To get analogous estimate for the maximal scale $r_{\max}$ is not so easy. This requires the exact knowledge of the total matter density Ω_{tot} for the homogeneous background and the knowledge of the baryon fraction Ω_b which gives $r_{\max}/r_{\min} \sim \Omega_{tot}/\Omega_b$ (where $\Omega = \rho/\rho_{cr}$ and ρ_{cr} is the critical density).

If we accept the value $\Omega_{tot} \sim 1$ (which is predicted by inflationary scenarios) and take the upper value for baryons $\Omega_b \lesssim 0.03$ (which comes from the primordial nuclearsynthesis), we find $r_{\max}/r_{\min} \gtrsim 30$. Another estimate can be found from restrictions on parameters of inflationary scenarios. Indeed, in inflationary models correct values for density perturbations give the upper boundary for the mass of the scalar field $m \lesssim 10^{-5}m_{Pl}$ which gives $r_{\max}/r_{\min} \gtrsim 10^5 T_*/m_{pl}$, where T_* is the critical temperature at which topology has been tempered. All these estimates are model dependent, however.

We also note that the more complex ground state can correspond to the situation in which the topological structure is not homogeneous (which requires studying the topology formation processes in the early Universe). In this case the minimal scale can possess some variation in space. Such variations however should have a characteristic wavelengths $\geq \ell_0$ (at the moment when topology has been tempered ℓ_0 corresponds to the horizon size), while the structural function should be described in a mixed representation $N_k\left(x\right)$, with $k > 2\pi/\ell_0$. In other words, the exact behavior of the structural function should be derived from observations.

The violation of the gravity law (the logarithmic behavior in a range of scales) has rather important consequences for the structure formation of the Universe. It turns out that in the range $r_{\min} \lesssim r \lesssim r_{\max}$ (in the case $r_{\max}/r_{\min} \gg 1$) the homogeneous distribution of matter is unstable [14]. And this instability works from the very beginning of the evolution of the Universe. To illustrate the existence of such an instability we consider the case when the maximal scale is absent $r_{\max} \to \infty$, or at least $r_{\max} \gg R_H$, where R_H is the Hubble radius. The number of baryons contained within a ball of a radius r in the case of a uniform distribution with a density n_b is given by $N_b\left(r\right) \sim n_b r^3$, while the dynamical mass or charge of every baryon increase

according to (57) as $q_b(r) \sim q_b r/r_{\min}$ (q_b is the proton mass m_p or the electric charge e). For the total dynamical mass and charge contained within the radius r we get

$$Q_{dyn}(r) = q_b(r) N_b(r) + \delta Q(r) \tag{58}$$

where $\delta Q(r)$ accounts for the contribution of baryons from the outer region[1] which does exist according to (56). Thus, for the dynamical mass we find $M_{dyn}(r) \geq \rho_b r^4/r_{\min}$, where $\rho_b = m_p n_b$. This means that the lower limit for the total dynamical density increases with the radius $\rho_{dyn} \sim M_{dyn}/r^3 \geq \rho_b r/r_{\min}$ and for sufficiently large $r \sim r_{cr}$ it will reach the value $\rho_b r_{cr}/r_{\min} = \rho_{cr}$, i.e., $\Omega_{tot} > 1$ and the ball of the radius r_{cr} starts to collapse (i.e., such a Universe must correspond to a closed cosmological model).

Consider now a small perturbation in the distribution of baryons. From (58) we find that the contribution of the perturbation in the total density will acquire the factor $\geq L/r_{\min}$, where L is the characteristic size of the perturbed region. For perturbations with $L/r_{\min} \gg 1$ this increase is considerable[2] and this will cause the decay of any initial homogeneous distribution which will last until baryons form a new stable equilibrium distribution.

It turns out that the stable equilibrium distribution of baryons is reached by a fractal law $N(r) \sim r^D$ with $D \approx 2$. Indeed, let us suppose that every baryon remains a point-like particle, while the contribution of its halo to the total dynamical mass and charge can be accounted for by the presence of some additional number of particles (fictitious particles). Then we can consider the "dynamical" (or effective) number of particles $\widetilde{N}_b = Q_{dyn}(r)/q_b$ and the stable equilibrium state will correspond to the homogeneous distribution of the dynamical charge and mass densities, i.e., $\widetilde{n}_b \sim \widetilde{N}_b/r^3 = const$, while the actual number of baryons in the range $r_{\min} \lesssim r \lesssim r_{\max}$, as it can be seen from (58), will follow the law $N_b(r) \leq Q_{dyn}(r)/q(r) \sim n_b r^2 r_{\min}$. In particular, this equilibrium state is consistent with the observed homogeneity of the Universe and the absence of large $\Delta T/T$ (the presence of fictitious baryons is essential here, for they give at the moment of recombination $T^3 \sim \widetilde{n} = const$), and it perfectly fits the observed galaxy distribution (see Ref. [11]). Above the scale $r_{\max}$ the law $1/r$ restores and the actual distribution of baryons has to cross-over to homogeneity $N_b(r) \sim r^3$. However the dynamical number of baryons will be $\widetilde{N}_b \sim N_b r_{\max}/r_{\min}$. The results of Ref. [11] provide that the size of the upper cutoff $r_{\max}$, if it really exist, must be more than $200Mpc$ and, therefore, for the actual baryon fraction we get a new estimate $\Omega_{tot}/\Omega_b \sim r_{\max}/r_{\min} \geq 10^5$. We also recall that, in fact, due to the large amount of fictitious baryons which have the homogeneous distribution, the observational limits on $\Delta T/T$ does not set any restrictions on possible values of $r_{\max}$ and it can be larger than the Hubble radius.

[1] In the case of homogeneous distribution of baryons this contribution gives $Q_{dyn}(r) \simeq r_{\max}/r_{\min}$ $q_b N_b(r)$ and the total dynamical mass is divergent in the limit $r_{\max} \to \infty$.

[2] In fact the total increase will be $\sim r_{\max}/r_{\min}$.

The instability of a homogeneous distribution of particles means that in thermodynamic equilibrium baryons were distributed with the fractal law (in particular, this is the only possibility when $r_{\max} \to \infty$), which gives a homogeneous distribution for the total dynamical mass and charge (baryons plus fictitious particles) densities. And the observed large scale structure of the Universe is nothing more than a remnant of the primordial thermodynamic equilibrium state (which is still in equilibrium). Upon the recombination, the homogeneous background charge disappears and the remnant of this process is seen now as the isotropic CMB radiation.

There exists one more argument in favour of this picture. It is the presence of a diffuse component of the X-ray background [39]. Indeed, some fraction of baryons is, at present, in the hot X-ray emitting intracluster gas that traces the primordial fractal distribution of baryons. Therefore, the total dynamical charge (baryons plus fictitious particles) forms a homogeneous background (i.e., homogeneous hot plasma) in space. In this sense the X-ray background has very similar origin as that of the cosmic microwave background radiation.

In this manner we see that dark matter, which in the above picture is represented by fictitious particles, is actually not dark, for it contributes to the X-ray background and CMB. Therefore, we should ask whether fictitious particles can be considered as real particles. The fictitious character of such particles displays itself in the fact that they show up only at scales $\ell > r_{\min}$ and, besides, they do not introduce additional degrees of freedom. E.g., density perturbations of fictitious particles are caused only by perturbations in the actual baryon density. We note that at scales $\ell > r_{\min}$ an arbitrary fluctuation in the baryon density is enhanced by fictitious particles up to the factor $\sim r_{\max}/r_{\min}$ and, therefore, observed $\Delta T/T$ can be caused by primeval thermal fluctuations. This also means that at scales $\ell > r_{\min}$ the standard dust-like equation of state for baryons is invalid (for baryons are actually not free but strongly interact) and we should expect the appearence of new phenomena, e.g., this may be used to explain the so-called dark energy (or quintessence) problem.There is also the possibility to attract fictitious particles to explain origin of galactic magnetic fields by the standard mechanism [40]. However these problems require a deeper investigation.

The renormalization of all interaction constants means that analogous background dynamical densities should exist for charges of all sorts. The concept of a homogeneous background gauge charge is not new, but it has been already suggested in particle physics by Higgs [41]. In the next section we discuss how such densities form the rest mass spectrum of elementary particles.

10 Origin of the rest mass spectrum of elementary particles

As it was pointed out above, the renormalization of interaction constants leaves a rather important imprint in particle physics. We show here that MOFT allows to

relate observed rest mass spectrum of elementary particles to cosmological parameters (background charge densities), and this can be used to get an independent estimate for the maximal scale $r_{\max}$. All particles are supposed to be massless on the very fundamental level. In the vacuum state the nontrivial topological structure of space produces some very small values for rest masses of particles (of the order $m_0 \sim k_{\max} = 2\pi/r_{\min}$, e.g., see Ref. [42]) and renormalizes the naked charge values $g \to \widehat{g}$ (for Fourier transforms we get $\widetilde{g}\,(k) = \sqrt{N_k} g$). The present Universe, however, is not in the vacuum state but can be characterized by a stable equilibrium state which is described by some background distribution of charges. Then the interaction of elementary particles with such a background forms the observed rest mass spectrum. We note that in the standard field theory such masses are still very small and this is the reason of why we need to attract additional scalar Higgs fields [41] (Higgs fields carry gauge charges and, therefore, a constant field produces a homogeneous background of a gauge charge density). In MOFT, however, interaction constants increase considerably with scales (e.g., in the region $r_{\min} \le r \le r_{\max}$ the dynamical charge behaves as $q\,(r) \sim q\ r/r_{\min}$) and, therefore, masses obtained can be close to observed values.

Indeed, in the case of fermions the field equation is $\left(\gamma^i\left(\partial_i + gA_i\right) + m_F\right)\psi = 0$, where γ^i is the Dirac matix, m_F is the naked value for the mass of a fermion, and A_i and ψ should be understood as generalized fields. Then the interaction with a background gauge field A forms the rest mass value $\delta m_F^2 = \langle \widehat{g}A\widehat{g}A\rangle \sim g^2\widetilde{n}_A/m_A^0$, where g is the naked value of the gauge charge, $m_A^0 \sim k_{\max}$ is the vacuum (or naked) value of the rest mass for gauge bosons, and $\widetilde{n}$ is the effective "dynamical" density of the background bosons which accounts for the renormalization of the charge (by the factor $\sqrt{N_k}$) (or the density of fictitious particles). It is important that the real density of the bosons does not coincides with $\widetilde{n}_A$ and is much smaller than it. At scales $r \gg r_{\max}$ (i.e., $k \ll k_{\min}$) the fractal picture crosses over to homogeneity and we can consider bosons to have a homogeneous distribution with a density n_A, while the total increase of the charge is $\widehat{g}^2 \sim g^2\ r_{\max}/r_{\min}$ and, therefore, we get for the rest mass the estimate $m_F^2 \sim g^2 n_A\ \left(r_{\max}/r_{\min}\right)/m_A^0$. Thus, we see that the observed tiny value of the gauge boson density (e.g., in the case of the electromagnetic field $n_\gamma \sim T_\gamma^3$, where T_γ is the CMB temperature) increases by the large factor $r_{\max}/r_{\min}$. We point out that the actual rest mass values depend on the densities of fictitious particles $\widetilde{n}_A$ which are not direct observables (on the contrary to n_A). In particular, in the case when the maximal scale is absent ($r_{\max} \to \infty$) the actual particles are distributed with the fractal law $N_A\,(r) \sim r^D$ which works for arbitrary large distances and the mean density vanishes $n_A \to 0$. However, we still can assign specific finite values to the densities of fictitious particles $\widetilde{n}_A$.

The actual value for the rest mass of a fermion depends on the number of different interactions (gauge fields) it is involved in, for all interactions produce independent contributions to the rest mass. E.g., neutrinos are the lightest particles, for they are involved in the weak interaction only, the electron rest mass forms mostly due

to the electromagnetic interactions, while the leading contribution to the rest mass of baryons (protons and neutrons) comes from the strong interactions. And indeed, the observed ratios of respective masses give roughly ratios of gauge charges.

In the case of gauge bosons there exist two independent contributions to the rest mass. The first comes from the self-interaction (for non-abelian fields), i.e., from the interaction of bosons with its own background and this contribution coincides with that for fermions. Thus, if such a contribution were alone the rest mass spectrum in the case of bosons would not be different from that of fermions. However, bosons interact as well with the background formed by fermions (e.g., in the case of photons only this kind of contribution does exist) and this deviates the resulting spectrum. E.g., in the case of photons the existence of the background hot X-ray emitting plasma gives rise to a non-vanishing photon rest mass $m_\gamma^2 \sim 4\pi e^2 \widetilde{n}_e/m_e$ (which is the Langmuir frequency), where $\widetilde{n}_e = \widetilde{n}_p$ is the background density of fictitious electrons.

We see that in MOFT the background densities of fictitious particles play the role of Higgs fields. In this picture Higgs fields represent phenomenological fields which account for the presence of the background gauge charge and this explains why we do not see Higgs particles in laboratory experiments. The fictitious particle number densities $\widetilde{n}_i$ include the renormalization of charge values (the contribution from halos) and, therefore, are not direct observables (by the astrophysical means of course; we do observe the rest mass spectrum). This means that the theory includes the same number of parameters to get the correct rest mass spectrum as the standard model in particle physics does (we mean here the number of essential parameters, for such parameters as masses of Higgs bosons are basically free parameters and are not fixed in the standard model). However in MOFT such parameters acquire a new astrophysical status.

Now, to make evaluations clear we consider as an example a real scalar field φ. Consider the expansion of the field φ in plane waves,

$$\varphi(x) = \sum_k \left(2\omega_k L^3\right)^{-1/2} \left(a_k e^{ikx} + a_k^+ e^{-ikx}\right), \tag{59}$$

where $\omega_k = \sqrt{k^2 + m_0^2}$ and m_0 is a naked value for the rest mass of scalar particles. In MOFT the field $\varphi(x)$ represents a generalized field which means that the creation and annihilation operators a_k^+ and a_k obey the relations (5). In particular, for the vacuum state we get (e.g., see (10))

$$\langle 0, N_k | \, a_k a_i^+ \, | N_k, 0 \rangle = n_{ik} = N_k \delta_{ki},$$

hence we find for fluctuations of the field potentials in the vacuum state

$$\langle \varphi(x)\, \varphi(x+r) \rangle = \frac{1}{(2\pi)^2} \int_0^\infty \frac{dk}{\omega_k} \frac{\sin kr}{kr} \Phi^2(k), \tag{60}$$

where $\Phi^2(k) = k^2 N_k$.

Let the potential of the scalar field be $V = \frac{\lambda}{4!}\varphi^4$. Then the observed value of the rest mass of scalar particles will be $m_{ph}^2 = m_0^2 + \frac{\lambda}{2}\left\langle \varphi^2(x)\right\rangle_{reg}$, where m_0^2 is the initial mass value (which corresponds to the trivial topology case $N_k = 1$) and $\left\langle \varphi^2(x)\right\rangle_{reg}$ is the regularized mean value $\left\langle \varphi^2(x)\right\rangle_{reg} = \langle 0, N_k|\, \varphi^2(x)\,|N_k, 0\rangle - \langle 0,1|\, \varphi^2(x)\,|1,0\rangle$. Consider the case where the maximal scale is absent ($k_{\min} \to 0$) and $m_0 = 0$ (or at least $m_0 \ll k_{\max}$). Then we can use the expression $N_k = 1 + [k_{\max}/k]$ and this gives

$$\delta m^2 = \frac{\lambda}{4}\frac{k_{\max}^2}{(2\pi)^2}\xi(2),$$

where $\xi(2) = \sum_{n=1}^{\infty} 1/n^2$. Thus, we see that in the non-trivial topology vacuum state, scalar particles acquire the rest mass of order $m \sim k_{\max}$ and, therefore, in what follows we can assume $m_0 > k_{\max}$.

The contribution from a background of scalar particles depends on specific realization of the generalized statistics for particles. In the simplest case we can accept expressions (34) which gives for the spectral density of fluctuations

$$\Phi^2(k) = k^2 N_k (1 + 2n_k), \tag{61}$$

where n_k is the standard occupation numbers for scalar particles. The case $N_k = 1$ corresponds to the ordinary field theory and the respective contribution n is known to be very small. Therefore, in what follows we neglect the corresponding term in (61). Then $N_k - 1 \approx 0$ as $k > k_{\max}$ and the integral in (60) can be cut off by $k = k_{\max}$. It follows that in this integral we can replace $\omega_k \simeq m_0$. Thus, we get for the nontrivial topology correction the value

$$\delta m_{top}^2 = \frac{\lambda}{2}\frac{\widetilde{n}}{m_0},$$

where $\widetilde{n}$ is the density of fictitious scalar particles which accounts for the renormalization of the charge

$$\widetilde{n} = \frac{1}{2\pi^2}\int_0^{k_{\max}} k^2 \left[k_{\max}/k\right] n_k dk.$$

For estimates we recall that according to (40) the ground state contains also hidden scalar particles which in the case $k_{\min} = 0$ gives as $k \to 0$, $n_k^* \sim (k_{\max}/k)^2$ and $n_k^* = 0$ as $k > k_{\max}$. In this case the integral (**??**) is divergent $\widetilde{n} \to \infty$ and if for estimates we accept $n_k \sim (k_{\max}/k)^{2-\delta}$ ($\delta \ll 1$) as $k < k_{\max}$, then we find that $n \sim k_{\max}^3$, while $\widetilde{n} \sim k_{\max}^3/\delta \gg n$, i.e., $\widetilde{n}$ can take arbitrary large values as $\delta \to 0$.

11 Effective dimension of the Universe

The growth of the spectral number of fields which takes place in the range of wave numbers $k_{\min}(t) < k < k_{\max}(t)$ leads to the fact that in this range our Universe

has to demonstrate nontrivial geometric properties. Indeed, as it was shown in the previous sections in the same range of scales a stable equilibrium distribution of baryons requires a fractal behavior with dimension $D \approx 2$, while the Newton's and Coulomb's energies of interaction between point particles show the logarithmic behavior. We recall that the logarithmic potential $\ln(r)$ gives the solution of the Poisson equation with a point source for two dimensions. Both these phenomena are in agreement with the observed picture of the Universe and it turns out that they have a common pure geometrical interpretation. Namely, we can say that in the range of scales $r_{\min} < r < r_{\max}$ (where $r_{\min} = 2\pi/k_{\max}$) some kind of reduction of the dimension of space happens.

Indeed, the simplest way to demonstrate this is to compare the spectral number of modes in the interval between k and $k + dk$ in MOFT, which is given by the measure $N_k d^3k/(2\pi)^3$, with the spectral number of modes for n dimensions in the standard field theory $d^nk/(2\pi)^n$. Hence we can define the effective dimension D of space as follows

$$k^3 N_k \sim k^D. \tag{62}$$

In the standard field theory $N_k = 1$ and we get $D = 3$, while in MOFT the properties of the function $N(k)$ were formed during the quantum period in the evolution of the Universe and depend on specific features of topology transformation processes. Thus, in general case, the effective number of dimensions D may take different values for different intervals of scales. If we take the value (48) we find that in the range of wave numbers $k_{\max} \geq k \geq k_{\min}$ (where the function N_k can be approximated by $N_k \sim k_{\max}/k$) the effective dimension of space is indeed $D \approx 2$. The scale $r_{\min} = 2\pi/k_{\max}$ can be called an effective scale of compactification of $3 - D$ dimensions, while the scale $r_{\max}$, if it really exists, represents the boundary after which the dimension $D = 3$ restores. We note also that after this scale the fractal picture of the Universe crosses over to homogeneity and the standard Newton's law restores.

In MOFT, N_k represents the operator of the number of fields (30) which is common for all types of fields and, therefore, plays the role of the density operator for the momentum space. We note that N_k is an ordinary function only in the case when topology changes are suppressed. In the coordinate representation the number of fields is described by an operator $N(x)$ (e.g., see the example of a one-dimensional crystal) which defines the density of physical space, i.e., the volume element is given by $dV = N(x)\, d^3x$.

Consider the relation between these two operators. In what follows we, for the sake of convenience, consider a box of the length L and will use the periodic boundary conditions (i.e., $\mathbf{k} = 2\pi\mathbf{n}/L$ and, as $L \to \infty$, $\sum_k \to \int (L/2\pi)^3\, d^3k$). From the dynamical point of view the operator N_k has a canonically conjugated variable $\vartheta(k)$ such that

$$[\vartheta(k), N_{k'}] = \vartheta_k N_{k'} - N_{k'}\vartheta_k = i\delta_{k,k'}\,. \tag{63}$$

These two operators can be used to define a new set of creation and annihilation

operators

$$\Psi^{+}(k)=\sqrt{N_k}e^{i\vartheta_k},\Psi(k)=e^{-i\vartheta_k}\sqrt{N_k} \tag{64}$$

which obey the standard commutation relations

$$\left[\Psi(k),\Psi^{+}\left(k'\right)\right]=\delta_{k,k'} \tag{65}$$

and have the meaning of the creation/annihilation operators for field modes (factorized over the number of particles). Thus the density operator for the momentum space is defined simply as $N_k=\Psi^{+}(k)\Psi(k)$.

In the case when topology transformations are suppressed the operator Ψ can be considered as a classical scalar field φ which characterizes the density of physical space and, in general, depends on time and space coordinates. We stress that this field has no a direct relation to Higgs fields discussed in the previous section. In what follows we consider the spatial dependence only, while the time dependence can be found in the same way. Indeed, in applying to the ground state $\Phi_0=|N_k\rangle$ (which is expressed by the occupation numbers (38) and (41)) the operators Ψ and Ψ^{+} change the number of modes by one, while the total number of modes $N\rightarrow\infty$. In this sense the state $\Phi_0'=\Psi\Phi_0\approx\Phi_0$. Thus, the classical field φ may be defined by relations

$$\varphi(k)=\langle N_k-1\left|\Psi(k)\right|N_k\rangle,\ \varphi^{*}(k)=\left\langle N_k\left|\Psi^{+}(k)\right|N_k-1\right\rangle, \tag{66}$$

which gives $\varphi(k)=\varphi^{*}(k)=\sqrt{N_k}$. The coordinate dependence of this field can be found in the standard way (e.g., see Ref. [24])

$$\Psi(\mathbf{r})=e^{-i\mathbf{r}\widehat{\mathbf{P}}}\Psi(0)e^{i\mathbf{r}\widehat{\mathbf{P}}} \tag{67}$$

where $\widehat{\mathbf{P}}$ is the total momentum operator. If in states O_i and O_f the system possesses fixed momenta $\mathbf{P}_i$ and $\mathbf{P}_f$, then

$$\langle O_i\left|\Psi(\mathbf{r})\right|O_f\rangle=\exp\left(-i\mathbf{r}\mathbf{k}_{if}\right)\langle O_i\left|\Psi(0)\right|O_f\rangle \tag{68}$$

where $\mathbf{k}_{if}=\mathbf{P}_i-\mathbf{P}_f$. We note that the creation/annihilation of a single mode is accompanied with the increase/decrease in the number of "dark" particles (40) by N_k and hence in the total momentum by $\mathbf{P}_k=zN_k\mathbf{k}$, where z is the number of different types of bosonic fields. Thus, from (68) and (66) we find that coordinate dependence will be described by the sum of the type

$$\varphi(x)\sim\sum_k c_k\exp\left(-i\mathbf{P}_k\mathbf{r}\right). \tag{69}$$

where $c_k\sim\sqrt{N_k}$. In the range of wave numbers $k_{\max}\geq k\gg k_{\min}$ we can use the approximation $N_k\simeq 1+k_{\max}/k$ and let the length of the box be $L<r_{\max}=2\pi/k_{\min}$.

Then we find that in the sum (69) the momentum $|\mathbf{P}_k| \geq zk_{\max}$ and, therefore, the maximal wavelength is indeed restricted by

$$\ell \leq \ell_0 < L, \tag{70}$$

where $\ell_0 = 2\pi/zk_{\max} = r_{\min}/z$, which means that at least in one direction the space box is effectively compactified to the size ℓ_0. From the physical standpoint such a compactification will be displayed in irregularities of the function $N(x) \sim |\varphi(x)|^2$ (we point out that the time dependence will randomize phases in (69)). E.g., the function $N(x)$ may take considerable values $N(x) \sim < N >$ only on thin two dimensional surfaces of the width $\sim \ell_0$ and rapidly decay outside, which may represent another view on our explanation of the formation of the observed fractal distribution of galaxies and the logarithmic behavior of Newton's potential. When considering the box of the larger and larger size $L \gg r_{\max}$ we find that $N_k \to N_{\max} \sim r_{\max}/r_{\min}$ and the effect of the compactification disappears (no restrictions on possible values of wavelengths emerge). Thus at scales $\ell > r_{\max}$ dimension $D = 3$ restores which restores both the standard Newton's law and the homogeneous distribution of galaxies.

12 Conclusions

We have shown that MOFT predicts a rather interesting physics for the range of scales $r_{\min} < r < r_{\max}$. First of all we point out that in this range the Universe acquires features of a two-dimensional space whose distribution in the observed 3-dimensional volume has an irregular character. This provides a natural explanation to the observed fractal distribution of galaxies and dark matter, which can be described by the presence of fictitious particles. Fictitious particles carry charges of all sorts and may be responsible for the origin of Higgs fields and the formation of the diffuse X-ray background. We recall that such properties originate from a primordial thermodynamically equilibrium state and are in agreement with the homogeneity and isotropy of the Universe. Thus, such a picture of the Universe represents a homogeneous background, while gravitational potential fluctuations should be considered in the same way as in the standard model. Nevertheless, we can state that in the range of wavelengths $r_{\min} < \lambda < r_{\max}$ the propagation of perturbations will correspond to the 2-dimensional law.

The fictitious character of particles which compose dark matter leads to strong correlations between perturbations in the actual baryon density and perturbations in dark matter. In particular, this also means that in the range of scales $r_{\min} < r < r_{\max}$ the standard dust-like equation of state for baryons is invalid and we might expect that this could allow to explain the so-called dark energy (or quintessence) phenomenon.

We note that Modified Field Theory represents nothing more but the standard field theory extended to the case where the topology of space is nontrivial and, in

general, may change. In this sense, from the qualitative standpoint predictions of MOFT are inevitable. If we believe that in the early Universe topology changes did occur, we merely have to observe now effects related to dark matter or fictitious particles. In the present paper we have shown that MOFT is a self-consistent, from particle physics standpoint, theory which can be readily used in applied problems. Yet, in the present form it is not exact. It requires , in the first place, an explicit description of topology transformations. There is no formal mathematical problems though (an appropriate term can be easily added to the Hamiltonian), however an adequate model for such processes, with a clear physical interpretation, is still missing. The importance of this problem is based on the observation that at very small scales ($\ell \leq \ell_{Pl}$) topology transformations still take place and those should define the behavior of the spectral number of fields N_k in the limit $k \to \infty$, i.e., the structure of space and the effective dimension at small scales. It is clear that in the exact theory ultraviolet divergencies should be absent, which means that integrals of the type (60) as $r \to 0$ should have finite values. This can be fulfilled if the spectral function is restricted by $N_k < k^{D-3}$ as $k \to \infty$ with $D < 1$ (i.e., at small scales the effective dimension is less than 1 or, in other words, the space disintegrates).

The situation is somewhat better with cosmological production of the field number density in the very early Universe. In this case there exists at least a clear qualitative model which can be used to illustrate the process, e.g. see. Ref. [43]. That model is based on the fact that near the singularity inhomogeneities of the metric have always a large scale character (i.e., $\ell_{in} \gg \ell_h \sim 1/t$, e.g. see, Ref. [44]) and this was shown to be valid during the quantum stage up to the moment when the classical space emerges (the moment of the origin of the classical space corresponds to $\ell_{in} \sim \ell_h$ [45]). Thus, every region of space of the size $L \lesssim \ell_h$ can be described in the leading order by a homogeneous mixmaster model. We can say that near the singularity the Universe splits into a set of homogeneous models, with parameters of the models varying in space. Then, to describe the topology formation process we can use the so-called third quantization scheme [46, 47, 48, 49] for every such model. In this scheme the number of universes $N_i(x)$ in a quantum state i will correspond to the number of fields in a region x of the size L. In general, the number of universes (or fields) $N_i(x)$ produced depends essentially on the choice of the initial quantum state. However, there exists a region of initial states which gives rise to the thermal distribution with $T \sim T_{pl}$ (e.g., see Refs. [47, 49]). Thus, there is always a room to speculate on the most natural initial conditions, the measure of initial conditions (and other stupid things which prove nothing but sound significant).

In the present paper we have supposed that upon the quantum period in the evolution of the Universe the spectral number of fields N_k is conserved, which means that N_k depends on time via only the cosmological shift of scales (i.e., $k(t) \sim 1/a(t)$, where $a(t)$ is the scale factor). However, there are reasonable arguments which show that the number of fields should eventually decay. Indeed, if we take into account thermal corrections to N_k, then instead of (48) we get the expression of the type

$N_k \sim 1 + T_*/k + ...$ In this case the temperature T_* plays the role of the maximal wave number $k_{\max}$. Why at present this temperature is so small $T_* \ll T_\gamma$ (where T_γ is the CMB temperature) requires a separate explanation. The situation is different when we allow the number of modes to decay. Phenomenologically, the decay can be described by the expression of the type $N_k \sim 1 + \left(N_k^0 - 1\right) e^{-\Gamma_k t}$ (where Γ_k is the period of the half-decay which, in general, can depend on wave numbers k). In the simplest case $\Gamma_k = const$ and this would lead to an additional monotonic increase of the minimal scale $r_{\min} \sim a\left(t\right) e^{\Gamma t}$, while the maximal scale changes according to the cosmological shift only $r_{\max} \sim a\left(t\right)$. We note that if the ground state of fields Φ_0 contains hidden bosons (40), the decay of modes will transform them into real particles and, therefore, the decay will be accompanied by an additional reheating which should change the temperature of the primordial plasma (which will produce an additional difference between T_* and T_γ).

The additional increase in $r_{\min}$ means that values of all interaction constants and rest masses of particles decrease with time, for at scales $\ell \gg r_{\max}$ the renormalized values of interaction constants have the form $\widetilde{\alpha} \sim \alpha r_{\max}/r_{\min} \sim e^{-\Gamma t}$. This would support the Dirac hypothesis [50] and recently observed suspected variation of the fine structure constant at high red shifts [51, 52, 53].

We also point out to the violation of the Pauli principle for wavelengths $\lambda > r_{\min}$ (more than one fermion can have a wavelength λ). Such particles are located in the volume $\gtrsim r_{\min}^3$ and at laboratory scales the portion of states violating the statistics is extremely suppressed $P \lesssim \left(L/r_{\min}\right)^3$, where L is a characteristic spatial scale of a system under measurement (e.g., if as such a scale L we take the Earth radius, this factor will be still extremely small $P \sim 10^{-32}$).

Acknowledgement

I am grateful to F. Columbus for the invitation to write a paper for this volume, and to D. Turaev for useful discussions at all stages of the research and a partial collaboration. I am especially grateful to J. Scoville whose simple questions gave me a new look at the subject. I would also like to beg my pardon of those authors, whose results have direct relation to this research and were not cited (for I have only accidental access to the LANL archive and then I am too lazy to search it thoroughly).

Warning: For the last five years, my attempts to find a support for this research from Russian funds and institutions were all in vain. E.g., RFFI steadily refuses to support this research. The "official science" (or at least big guys in the Russian Academy of Science) seem to have some serious principle objections: perhaps, the results do not fit the striking, fascinating, predictive, solving all problems, etc., inflationary paradigm, or, may be, "...such subtleties had not been considered yet ...", and "... this is not what we expect and what we want from quantum gravity..."(Physical Review referees), or at last " ... starting from scales more than a few *Mpc* the Newton's law is well-verified ...", and " ... it is impossible to construct any reasonable homogeneous and isotropic model for the gravitational potential which

increases as $\ln r$ ..." (JETP Letters). In any case I feel it is necessary to warn young scientists about the situation (please, do not take this approach and all the results too seriously, for all those are not more but rubbish!).

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